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Markov Decision Chains and Hamiltonian Transition Matrices

Abstract

We consider the famous Hamiltonian cycle problem (HCP) embedded in a Markov decision process (MDP). More specifically, we consider a moving object on a graph G where, at each vertex, a controller may select an arc emanating from that vertex according to a probabilistic decision rule. A stationary policy is simply a control where these decision rules are time invariant. Such a policy induces a Markov chain on the vertices of the graph. Therefore, HCP is equivalent to a search for a stationary policy that induces a 0 - 1 probability transition matrix whose non-zero entries trace out a Hamiltonian cycle in the graph. We present some algebraic properties of a particular class of probability transition matrices, namely, Hamiltonian transition matrices. Each matrix P in this class corresponds to a Hamiltonian cycle in a given graph G on N nodes and to an irreducible, periodic, Markov chain. We show that a number of important matrices traditionally associated with Markov chains, namely, the stationary, fundamental, deviation and the hitting time matrix all have elegant expansions in the first N-1 powers of P, whose coefficients can be explicitly derived. We also consider the resolvent-like matrix associated with any given Hamiltonian cycle and its reverse cycle and demonstrate an interesting link with the classical Chebyshev polynomials of the second kind, along a path connecting these cycles in the space of occupational measures.